DYNAMIC ASSET ALLOCATION STRATEGIES ACROSS HEDGE FUND INDICES

The performance of various asset allocation strategies across hedge fund indices using alternative static as well as dynamic methods that are based on forecasts of conditional volatility is examined. Daily rebalanced dynamic portfolios are examined for the three main sub-indices of Standard & Poor’s Hedge Fund Index. Out-of-sample results are also shown for nine Credit Suisse First Boston / Tremont hedge fund indices. Time varying volatility and volatility clustering characterize most hedge fund indices. Using forecasts of next-period volatility based on time-varying procedure generally improves the risk-return profile of the portfolio. All of the dynamic hedge fund index portfolios largely outperform the passive S&P 500 index, both on an expected return and risk-adjusted return basis.

Introduction

Since the mid 1990’s, hedge funds have emerged as a popular investment vehicle for high net-worth individuals and institutional investors. They have also attracted considerable interest from academics. The tremendous popularity of this new investment vehicle can be explained by the highly diverse investment strategies employed by hedge fund managers and their alleged heterogeneous return profiles.

The investable hedge fund indices which have recently appeared, such as the CSFB/Tremont Sector Indices, provide an opportunity to easily exploit tactical asset allocation strategies in the alternative assets space. Funds of Funds (FOF’s), Pension Funds, Endowments, Family Funds and other institutional investors have taken substantial positions in investable hedge funds. The work herein proposes dynamic asset allocation strategies to hedge fund indices based on the minimum variance and the maximum Sharpe ratio approaches. Such strategies should be of great interest to FOF’s looking to optimize their portfolios through time. This paper extends the static mean-variance asset allocation framework to allow for time varying volatility of

---

1 As of mid-2005, hedge fund assets overseen by single managers amounted to $1.371 trillion, whereas the holdings in funds of hedge funds rose to $709 billion, according to Hedge Fund Manager Magazine.
returns. The result is a dynamic optimization framework, which we apply to the FOF problem of asset allocation across hedge funds.

This paper is the first that we are aware of to explicitly account for time varying volatility in the construction of dynamic optimal portfolios of hedge funds, including, where appropriate, a multivariate asymmetric GARCH model for conditional volatility forecasting for optimal hedge fund indices asset allocation.

The approach allows to directly account for skewness and kurtosis of returns. Furthermore, the asymmetric GARCH framework allows for leverage effects, whereby negative return shocks could exacerbate conditional volatility.

Data Description

To represent the style-based investment strategies in an alternative investment universe, two of the most prominent hedge fund index providers are selected: Credit Suisse First Boston/Tremont Hedge Fund Indices (CSFB/T HF Indices) and Standard and Poor’s Hedge Fund Indices (S&P HF Indices). These indices have several advantages relative to competitors in terms of both calculation ease and transparency.

Over the period examined (1994-2006), eight of the nine CSFB/T hedge fund indices outperform the S&P 500 benchmark on a risk-adjusted basis (Sharpe Ratio). The best risk-adjusted return is achieved by the equity market neutral hedge fund index, with an annualized mean return of 10.07% and a Sharpe ratio of 3.43. Event driven and convertible arbitrage indices are next in rank, with Sharpe ratios of 2.08 and 1.88, respectively. The worst-performing, and the only hedge fund index which underperforms the S&P 500 benchmark, is the dedicated short bias (Sharpe ratio of -0.06).

We also find that the equity market neutral fund is highly correlated with the most of the other indices, with the exception of the global macro and managed futures series.

The Standard and Poor’s Hedge Fund Index was launched in October 2002. The index is equally-weighed across various alternative investment strategies and is re-balanced annually. The distinctive characteristic of this index is the availability of daily returns data. The main S&P Hedge Fund Index consists of three (style) Indices that are deemed to broadly represent the hedge fund investing universe: arbitrage, event-driven and directional/tactical. Each strategy in turn consists of three underlying strategy components. The arbitrage index includes equity market neutral, fixed income arbitrage and convertible arbitrage. The event-driven index includes merger arbitrage, distressed situations, and special situations. The directional/tactical index incorporates equity long/short, managed futures, and global macro.

The main S&P Hedge Fund Index is an index suitable for dynamic asset allocation. Constituent strategies however cannot be invested in on a stand-alone basis. Thus, the results of the analysis conducted on weekly and daily data using three constituent strategies of the S&P Hedge Fund Index are not replicable using a single tradable investment portfolio. Nevertheless, examination of dynamic/tactical asset allocation strategies with weekly and daily re-balancing horizons serves to complement the results obtained using the monthly re-balancing strategies with

---

2 See [http://www2.standardandpoors.com/spf/pdf/index/Hedge_AIQ1.pdf](http://www2.standardandpoors.com/spf/pdf/index/Hedge_AIQ1.pdf) for a discussion of the construction of these indices.
CSFB/T Indices. They also serve as a proxy for the expected characteristics of strategy returns for
week.

For the period analyzed, all three hedge fund indices strongly outperformed the S&P 500
Index, on a risk adjusted basis (Sharpe ratio), with the event driven, directional/tactical and
arbitrage indices generating the best performance.

Methodology

Portfolio Construction based on Maximum Sharpe Ratios

The simplest dynamic hedge fund indices portfolios considered in this work are based on
standard mean-variance Markowitz optimization. Past returns, volatilities and cross-correlations
serve as inputs in the estimation of the next-period efficient frontier. Maximum Sharpe ratio
portfolios are constructed for monthly and weekly hedge fund indices data.

Portfolio Construction based on Past Volatility

In order to construct portfolios based on historical volatility, the weights of each hedge
fund index within the next period portfolios need to be computed. A Global Minimum Variance
(GMV) asset allocation approach is used in this work. Thus the optimal weights \( \omega_i \) depend on
the predicted covariance matrix \( H_{t+1} \).

Assuming a diagonal covariance matrix for nine univariate CSFB/T Hedge Fund Indices, the weights of the univariate diagonal portfolio are given by:

\[
\omega_{t,i} = \frac{\hat{\sigma}^{-2}_{t+1,i}}{\sum_{j=1}^{9} \hat{\sigma}^{-2}_{t+1,j}}
\]

where for CSFB/T indices \( i = 1,2,3,...,9 \) and for S&P indices \( i = 1,2,3 \). \( \hat{\sigma}^{2}_{t+1,i} \) is the past variance
of the monthly returns of the \( i \) th CSFB/T Hedge Fund index or is the past variance of weekly or
daily returns of the \( i \) th S&P Hedge Fund Index. The dynamic variance is either forecasted by the
asymmetric univariate GJR-GARCH(1,1) model or estimated based on past volatility. The same
approach is used for finding the optimal weights of Standard and Poor’s Hedge Fund Indices for
weekly and daily rebalanced portfolios.

In addition to the univariate GJR-GARCH(1,1) Past Volatility risk estimates, multivariate
asymmetric GJR-GARCH(1,1) estimates are used for calculation of weights for daily-rebalanced
portfolios. The multivariate GJR-GARCH(1,1) portfolio forecast covariance matrix based on the
three S&P HF indices are then used to find optimal next-period index weights. Portfolio
optimization based on the Markowitz approach requires inputs of expected returns, variances and
cross-correlations to generate an efficient investment frontier. The performance of such a
portfolio critically depends on the quality of forecasts of the expected returns vector and the
covariance matrix. In this paper, next-day variances and cross-correlations are forecasted by the
Multivariate GJR-GARCH(1,1) model, whereas the expected returns are equal to the average
returns over the in-sample period.
Portfolio Construction based on ARCH/GARCH Conditional Volatility Estimation

As a first step in the construction of the portfolios, residuals from OLS estimation of returns are tested for ARCH (autoregressive conditional heteroscedasticity) behavior. To predict the volatilities of next-period returns, an Asymmetric GARCH model (GJR-GARCH) with t-distributed errors is used.

The GJR-GARCH specification is used to account for possible leverage effects, with negative shocks serving to enhance conditional volatility. Krokhmal, Uryasev, and Zrazhevsky (2002) and Favre and Signer (2002) state that assuming normality in hedge fund returns leads to portfolios that are more risky than in the case when asymmetry is accounted for. Conditional variances are parameterized by a GJR-GARCH model of orders p and q.

The GJR-GARCH(p,q) model is thus of the following form:

\[
\sigma_i^2 = \alpha_0 + \sum_{i=1}^{p} (\alpha_1 + \gamma_i S_i) e_{i-1}^2 + \sum_{i=1}^{q} \beta_i \sigma_{i-1}^2
\]

where \(S_i\) is a dummy variable for negative residuals, defined as:

\[
S_i = \begin{cases} 
1, & e_i < 0 \\
0, & e_i > 0 
\end{cases}
\]

Using the GJR-GARCH model, the next-day conditional volatility for monthly, weekly and daily-rebalanced hedge fund indices is then forecasted by:

\[
\hat{\sigma}_{i+1}^2 = \hat{\alpha}_0 + (\hat{\alpha}_1 + \hat{\gamma}_i S_i) e_i^2 + \hat{\beta}_i \sigma_i^2
\]

where \(S_i\) is a dummy variable for negative residuals, as defined in Equation (2).

A univariate GJR-GARCH(1,1) model with a BHHH (Berndt et al. (1974)) algorithm is estimated on the Hedge Fund Indices. For the S&P daily returns, 800 rolling in-sample observations are used to forecast volatilities for 143 out-of-sample days, from December 2005 until the end of June 2006. For the S&P weekly returns data, 157 rolling in-sample weekly observations are used to forecast volatilities for 31 out-of-sample weeks, from the beginning of December 2005 until the end of June 2006.

As expected, ARCH/GARCH terms are generally significant for the daily and weekly the S&P Hedge Fund Indexes. For the daily series, the asymmetric volatility term is negative and significant, consistent with leverage effects typically found for equity markets – with negative shocks in returns serving to enhance conditional volatility.

For the monthly CSFB/T Hedge Fund indices, 50 rolling windows of 100 observations are used in the estimation. January 1994 through March 2002 serves as an initial calibration period for subsequent volatility forecasts from April 2002 until June 2006. In the pre-tests,
ARCH/GARCH effects are only observed for the CFSB/T Market Neutral and Fixed Income series. Consequently, GARCH forecasts of volatility is only applied to these series.

In addition to the univariate GJR-GARCH(1,1) specification, a multivariate asymmetric GARCH model extending Switzer and El Khoury (2007) is applied for the daily S&P Hedge Fund Indices data, where ARCH/GARCH effects are identified. The added benefit of the Multivariate GARCH specification in dynamic asset allocation is that the covariance matrix is estimated jointly across assets, as opposed to inferred from the forecasts of the global minimum variance formula.

The covariance matrix of the multivariate asymmetric bivariate GARCH can be written as:

\[ H_t = C'C + AH_{t-1}A + B'e_t'e_t'B + G'\eta_{t-1}\eta_{t-1}'G \]  \hspace{1cm} (4)

where and G is a matrix of coefficients, and \(\eta_t\) is the additional quadratic form of the vector of negative return shock. \(H_t\) is a linear function of its own past values as well as of values of squared shocks. The inclusion of \(\eta_t\) in the above form not only accounts for asymmetry in the conditional variances but also allows for an asymmetric effect in the conditional covariance. This approach allows for time variation in the correlations across the various series.

Parameter estimates are obtained by maximizing the log-likelihood function. Conditional log-likelihood functions are computed as:

\[ L_t(\theta) = - \log 2\pi - \frac{1}{2} \log |H_t| - \frac{1}{2} e_t'(H_t(\theta)e_t(\theta)) \]  \hspace{1cm} (5)

where \(\theta\) is the vector of all parameters of the model. To maximize this log-likelihood function, we use the simplex and Berndt, Hall, Hall, and Hausman (1974) algorithms.

Benchmark Portfolio and Transactions Costs

Four investment portfolios are examined: the maximum Sharpe portfolios, Past Volatility portfolios, and GJR-GARCH(1,1) portfolios, and the benchmark S&P 500 Index. The latter is held as a passive portfolio. For the case of daily rebalancing, multivariate GJR-GARCH(1,1) portfolio replace the maximum Sharpe portfolio in the analysis.

We also incorporate transactions costs in the analysis. The results reported here assume transaction costs of 25 basis points. Comparative and lower levels have been used in prior academic works that looked into investment strategies for traditional asset classes and are believed to be appropriate for an alternative investment universe composed of “investable” hedge fund indices.

Results

CSFB/Tremont Monthly Rebalanced Portfolios

The performance of the CSFB/Tremont monthly rebalanced dynamic portfolio based on conditional volatility forecasting from GJR-GARCH(1,1) is compared to the Past Volatility portfolio and the S&P 500 Index.
The risk-adjusted performance of the portfolios under consideration (Maximum Sharpe, Past Volatility, Univariate GARCH and S&P500) are compared based on Sharpe Ratio, defined as the ratio of the annualized mean portfolio return to the annualized portfolio standard deviation:

$$SR_p = \frac{\mu_p}{\sigma_p}$$  \hspace{1cm} (5)

The out-of-sample testing period for the monthly analysis extends from May 2002 until June 2006, for a total of 50 return observations. To illustrate the approach, the optimal asset allocation weights to each of the nine CSFB/Tremont Hedge Fund Indices under consideration, as directed by the Univariate GJR-GARCH portfolio investment strategy is provided in Table 1 below. Optimal weights are shown for the months of January and the out-of-sample years are 2003, 2004, 2005 and 2006.

[Please insert Table 1 about here]

After accounting for transaction costs, based on the Sharpe Ratio rankings the Past Volatility portfolio ($SR_p=3.46$) performs as well as the Maximum Sharpe Ratio portfolio ($SR_p=3.44$) while the GJR-GARCH(1,1) portfolio dominates ($SR_p=3.53$). All three portfolios still largely outperform their benchmark S&P 500 Index ($SR_p=0.37$).

**Standard and Poor’s Weekly Rebalanced Portfolios**

For weekly data, after transactions costs are accounted for, GARCH (1,1) exhibits the best performance ($SR_p=5.07$), followed by Past Volatility ($SR_p=5.06$), the Maximum Sharpe Ratio ($SR_p=4.47$), and the S&P 500 Index ($SR_p=0.03$), respectively.

We also conducted the analysis using daily data. Ignoring transactions, costs, the risk-adjusted returns are markedly better for the Multivariate GARCH(1,1) model is shown to dominate ($SR_p=7.45$), followed by the univariate GARCH(1,1) model ($SR_p=7.30$), the Past Volatility model ($SR_p=6.20$) and the distant S&P 500 Index benchmark ($SR_p=0.32$). In general the annualized returns are higher for more actively managed portfolios.

The optimal weights for the Multivariate GARCH (1,1) model are provided in Table 2.

[Please insert Table 2 about here]

Figure 1 shows the wealth effects of the various portfolios based on these weights for Standard & Poor’s Hedge Fund Indices portfolios, before transaction costs are included.

[Please insert Figure 1 about here]

---

3 The Past Volatility Portfolio did not rank as high before transactions costs are taken into consideration. However, using transactions costs of 50 basis points alters the rankings: with the more frequent rebalancing of the GARCH portfolios, the Past Volatility portfolio dominates it, on a Sharpe ratio basis.
Of course, the benefits of more frequent rebalancing strategies are diminished by the higher trading costs, in terms of the risk adjusted returns. Nevertheless, when plausible levels of transactions costs are included, all of the active portfolios dominate the passive benchmarks.

Conclusion

This paper examines the return/risk benefits of portfolios of hedge fund indices with time varying volatility, and with returns distributions that are skewed and leptokurtotic. The results show that there are distinct benefits in volatility reduction for portfolios constructed based on conditional volatility forecasting relative to static portfolios including the S&P 500 benchmark. These results are robust to transactions costs.

For the S&P Hedge funds, portfolios constructed based on conditional volatility models that embody asymmetric volatility outperform on a risk-adjusted basis because of the larger returns, as opposed to a reduction in volatility, versus a portfolio structured based on the Past Volatility model.

References


<table>
<thead>
<tr>
<th></th>
<th>Convertible Arbitrage</th>
<th>Dedicated Short Bias</th>
<th>Emerging Markets</th>
<th>Equity Market Neutral</th>
<th>Event Driven</th>
<th>Fixed Income Arbitrage</th>
<th>Global Macro</th>
<th>Long Short Equity</th>
<th>Managed Futures</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 2006</td>
<td>8.94%</td>
<td>2.46%</td>
<td>2.94%</td>
<td>30.52%</td>
<td>7.39%</td>
<td>34.98%</td>
<td>4.91%</td>
<td>4.11%</td>
<td>3.75%</td>
</tr>
<tr>
<td>January 2005</td>
<td>11.72%</td>
<td>2.99%</td>
<td>3.45%</td>
<td>26.71%</td>
<td>8.93%</td>
<td>31.63%</td>
<td>5.16%</td>
<td>4.91%</td>
<td>4.50%</td>
</tr>
<tr>
<td>January 2004</td>
<td>7.82%</td>
<td>1.92%</td>
<td>2.18%</td>
<td>50.37%</td>
<td>5.79%</td>
<td>22.81%</td>
<td>3.01%</td>
<td>3.17%</td>
<td>2.94%</td>
</tr>
<tr>
<td>January 2003</td>
<td>14.22%</td>
<td>3.65%</td>
<td>3.86%</td>
<td>44.38%</td>
<td>10.73%</td>
<td>6.46%</td>
<td>5.36%</td>
<td>5.84%</td>
<td>5.52%</td>
</tr>
</tbody>
</table>

This table shows the optimal asset allocation weights to each of the nine CSFB/Tremont Hedge Fund Indices under consideration, as directed by the Univariate GJR-GARCH portfolio investment strategy. Optimal weights are shown for the months of January and the out-of-sample years are 2003, 2004, 2005 and 2006.
Table 2: Standard & Poor’s Hedge Fund Indices Univariate GJR-GARCH Portfolio Weekly Allocations as of the 1st trading day of each month (December 2005-June 2006)

<table>
<thead>
<tr>
<th></th>
<th>Event Driven</th>
<th>Directional/Tactical</th>
<th>Arbitrage</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 2006</td>
<td>29.84%</td>
<td>4.66%</td>
<td>65.50%</td>
</tr>
<tr>
<td>May 2006</td>
<td>47.56%</td>
<td>8.58%</td>
<td>43.86%</td>
</tr>
<tr>
<td>April 2006</td>
<td>49.96%</td>
<td>6.93%</td>
<td>43.11%</td>
</tr>
<tr>
<td>March 2006</td>
<td>44.71%</td>
<td>7.83%</td>
<td>47.46%</td>
</tr>
<tr>
<td>February 2006</td>
<td>29.97%</td>
<td>4.59%</td>
<td>65.44%</td>
</tr>
<tr>
<td>January 2006</td>
<td>58.59%</td>
<td>6.91%</td>
<td>34.50%</td>
</tr>
<tr>
<td>December 2005</td>
<td>30.28%</td>
<td>5.85%</td>
<td>63.87%</td>
</tr>
</tbody>
</table>

This table shows the optimal asset allocation weights to each of the three Standard & Poor’s Hedge Fund Indices under consideration, as directed by the Univariate GJR-GARCH portfolio investment strategy. Optimal weights are shown for the first week of the seven months between December 2005 and June 2006.
Figure 1: Out-of-Sample Wealth Effects of Weekly-Rebalanced Standard & Poor’s Hedge Fund Indices Portfolios, Before Transaction Costs are Included